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MULTIPLE FRAME SURVEYS I[1]

by
JOSE S. GUTIERREZ[2]

PART I: THEORY OF MULTIPLE FRAME SURVEYS

Introduction

In sample survey methodology one often finds that a frame known to cover approximately all units in the population is one in which one is costly while other frames (e.g., special lists of units) are available for cheaper sampling methods. However, the latter usually only cover an unknown or only approximately known fraction of the population.

The technique of multiple frame surveys has been used occasionally and under special situations. Two known applications of multiple frame surveys are:

(1) The 1960 Survey of Agriculture of the U.S. Bureau of the Census used two frames, viz., (a) a frame of farms conceptually and operationally associated with the A-1 listings of the 1959 Census of Agriculture; (b) a frame based on the conventional "area sampling" approach.

(2) The Statistical Laboratory, Iowa State University had used a two-frame approach in a small study of "Effects of Industrialization on Farming" which carried out for the Department of Economics and Sociology of the same university. The frames used were (a) The customary rural area frame for sam-

[1] Research Paper No. 68-JSG—3.
[2] Associate Professor, Statistical Center, U.P.

pling farm operators, and (b) Employees of a motor company who are also farm operators.

In the Philippines a private organization utilized a quite different technique of multiple frame survey. The survey utilized two frames, namely: (a) a list of coffee growers, and (b) a list of coffee dealers. This technique used in this survey will be discussed more in detail, later.

Meaning of Multiple Frame Surveys

To fix the idea of multiple frame surveys, consider two frames A and B and assume that a sample has been drawn from each frame. The sample designs may be completely different in the two frames but the following assumptions are made:

(1) Every unit in the population of interest belongs to at least one of the frames (exhaustive).

(2) It is possible to know for each sampled unit whether or not it belongs to the other frame.

This means the units of the sample can be grouped into three ($2^2 - 1$) domains:

DOMAIN	FRAME	EXPLANATION
a	A	The unit belongs to frame A.
b	B	The unit belongs to frame B.
ab	A and B	The unit belongs to both frames. Intersection or common to both frames.

With this conceptual division four different situations regarding the state of knowledge of the total number of units in the frames and in the domains and in our ability to allocate prescribed sample sizes to the domains can be enumerated:

Situation No.	Knowledge of population numbers in domains and frames	Possibility of fixed sample allocations to domains and frames	Nature of Domains
1	All domain sizes N_a, N_b, N_{ab}, etc. are known.	Feasible to allocate prescribed sample sizes to domains.	Domains ≡ Strata.
2	All domain sizes N_a, N_b, N_{ab}, etc. are known.	Prescribed sample sizes can only be allocated to frames.	Domains ≡ Post Strata.
3	Domain sizes are not known, but frame sizes are known.	Prescribed sample sizes can only be allocated to frames.	Domains ≡ domains proper.
	Neither domain sizes nor frame sizes are known.	Prescribed sample sizes can only be allocated to frames.	Domains ≡ domains in populations of unknown size.

A convenient notation for a two-frame surveys is as follows:

	Frame		Domain		
	A	B	a	b	ab
Population number	N_A	N_B	N_a	N_b	N_{ab}
Sample Number	n_A	n_B	n_a	n_b	n'_{ab}, $n'_{a'b}$
Population total	Y_A	Y_B	Y_a	Y_b	Y_{ab}
Population mean	$\bar{Y}_A$	$\bar{Y}_B$	$\bar{Y}_a$	$\bar{Y}_b$	$\bar{Y}_{ab}$
Sample total	y_A	y_B	y_a	y_b	y'_{ab} y'_{ab}
Sample mean	$\bar{y}_A$	$\bar{y}_B$	$\bar{y}_a$	$\bar{y}_b$	$\bar{y}'_{ab}$ $\bar{y}'_{ab}$
Cost of sampling unit	c_A	c_B			

Sample values apply to cases of drawing random samples from both frames; the quantities n'_{ab} and $n'_{a'b}$ denote respectively the subsamples of n_A and n_A respectively which fall into the overlap domain ab. The corresponding means $\bar{y}'_{ab}$, $\bar{y}'_{a'b}$ can only be computed if $n'_{ab} > 0$ and $n'_{a'b} > 0$.

Estimation

Optimum p for a fixed cost. The estimation of population totals and means in the means in the first situation is reduced to the standard methodolgy for stratified sampling. Two approaches leading to identical formulas are possible in situations 2 and 3, namely: (a) the theory of domain estimation, or (b) a special method of weight variables. For approach (b) the following attributes the units in the two frames:

$$\text{Frame (A)}: u_i = \begin{cases} y_i \text{ if } i^{th} \text{ unit is in domain (a)} \\ py_i \text{ if } i^{th} \text{ unit is in domain ab} \end{cases}$$

$$\text{Frame ( )}: u_i = \begin{cases} y_i \text{ if } i^{th} \text{ unit is in domain (b)} \\ qy_i \text{ if } i^{th} \text{ unit is in domain ab} \end{cases}$$

where p and q are two fixed numbers (which are optimally determined) with $p + q = 1$. In stratum A there are N_{ab} units carrying a characteristic $u_i = py_i$ and in stratum B there are N_{ab} units carrying the characteristic $u_i = qy_i$. Clearly, Y, the total of the y_i for the original population of $N = N_a + N_{ab} + N_b$ units, is equal to the sum of u_i, U for the new population $N^* = N_a + 2N_{ab} + N_b$ units since

$$Y = Y_a + Y_{ab} + Y_b = Y_A + pY_{ab} + qY_{ab} + Y_b = u.$$

The estimate of the population total of the Y characteristics is:

$$\hat{Y} = N_a \bar{y}_a - N_{ab} \cdot p \bar{y}'_{ab} + N_{ab} + N_b \bar{y}_b$$

$$= \frac{N_a}{n_a} (y_a + p y'_{ab}) + \frac{N_B}{n_B} (y_b + q y''_{ab})$$

with the following approximate variance (fpc ignored and for sufficiently large samples from both frames):

$$\text{Var } \hat{Y} = \frac{N^2_A}{n_A} \{ (1-\alpha) \sigma^2_b + \alpha p^2 \sigma^2_a + \alpha (1-\alpha) (\bar{y}_a - p \bar{y}_{ab})^2 \}$$

$$+ \frac{N^2_B}{n_B} \{ (1-\beta) \sigma^2_{ab} + \beta q^2 \sigma^2_{ab} + \beta (1-\beta) (\bar{y}_b q \bar{y}_{ab})^2 \}$$

$$+ \frac{N_A}{n_A (1\text{-}p)} \{ - (1\text{-}p) + (1\text{-}\alpha \frac{N_A}{N_B}) \left(\frac{\bar{y}_b - (1\text{-}p)\, \bar{y}_{ab}}{\sigma^2_{ab}} \quad \bar{y}_{ab} \right) \} = 0$$

Substituting the values of β, n_B and q, Equation 3 will yield the following relation

$$\{ p - \frac{(1\text{-}\alpha)\,(\bar{y}_a - p\,\bar{y}_{ab})\,\bar{y}_{ab}}{\sigma^2_{ab}} \} + \frac{1}{1\text{-}p} \{ - (1\text{-}p)$$

$$+ (1\text{-}\alpha \frac{N_A}{N_B}) \left(\frac{\bar{y}_b - (1\text{-}p)\, \ddot{y}_{ab}}{\sigma^2_{ab}} \bar{y}_{ab} \right) \}$$

$$= \frac{p\,\sigma^2_{ab} - (1\text{-}\alpha)\,(\bar{y}_a - p\,\bar{y}_{ab})\,\bar{y}_{ab}}{\sigma^2_{ab}}$$

$$+ \{ -1 + (1\text{-}\alpha \frac{N_A}{N_B}) \left(\frac{\bar{y}_b - (1\text{-}p)\,\bar{y}_{ab}}{(1\text{-}p)\,\sigma^2_{ab}} \ddot{y}_{ab} \right) \}$$

$$= p\,(1\text{-}p)\,\sigma^2_{ab} - (1\text{-}p)\,(1\text{-}\alpha)\,(\bar{y}_a - p\,\bar{y}_{ab})\,\bar{y}_{ab}$$

$$- (1\text{-}p)\,\sigma^2_{ab}\,(1\text{-}\alpha \frac{N_A}{N_B})\,[\bar{y}_B - (1\text{-}p)\,(\bar{y}_{ab})\,\bar{y}_{ab}] = 0$$

Writing as a quadratic of p, we obtain:

$$p^2 (-\sigma^2_{ab} - \bar{y}^2_{ab} + \alpha\,\bar{y}^2_{ab})$$

$$+ p\,(\sigma^2_{ab} + \bar{y}_a \bar{y}_{ab} - \alpha\,\bar{y}_a \bar{y}_{ab} + \sigma^2_{ab} \bar{y}_{ab} - \alpha\,\bar{y}^2_{ab} + \bar{y}^2_{ab} - \alpha \frac{N_A}{N_B} \bar{y}^2_{ab})$$

$$-\bar{y}_a \bar{y}_{ab} + \alpha\,\bar{y}_a \bar{y}_{ab} - \sigma^2_{ab} - \bar{y}_b \bar{y}_{ab} - \bar{y}^2_{ab} - \alpha \frac{N_A}{N_B} y_B y_{ab} + \alpha \frac{N_A}{N_B} y_{ab}) = 0$$

where $\alpha = \frac{N_{ab}}{N_A}$, $\beta = \frac{N_{ab}}{N_B} = \alpha \frac{N_A}{N_\beta}$.

The basic problem is the allocation of the sample to the different frames or the determination of the optimum value of p (or q). Minimizing the variance for a given cost ($C = c_A n_A + c_B n_B$), the following were obtained:

$$(1)\quad V_{n_A} = O: \left(\frac{N_A}{n_A}\right)^2 \left\{ \sigma^2_a (1\text{-}\alpha) + \sigma^2_{ab}\, p^2\, \alpha + \alpha\, (1\text{-}\alpha) (\bar{y}_a - p\bar{y}_{ab})^2 \right\} - \lambda\, c_A = O$$

$$(2)\quad V_{n_B} = O: \left(\frac{N_B}{n_B}\right)^2 \left\{ \sigma^2_b (1\text{-}\beta) + \sigma^2_{ab}\, q^2\, \beta + \beta\, (1\text{-}\beta) (\bar{y}_b - q\bar{y}_{ab})^2 \right\} - \lambda\, c_B = O$$

and

$$(3)\quad V_p = O: \frac{N^2_A}{n_A} \left\{ \alpha\, p - \frac{(1\text{-}\alpha)\, (\bar{y}_a - p\bar{y}_{ab})}{\sigma^2_{ab}}\, \bar{y}_{ab} \right\} + \frac{N^2_B}{n_B} \left\{ - q\, \beta + \frac{(1\text{-}\beta)\, (\bar{y}_b - q\bar{y}_{ab})}{\sigma^2_{ab}}\, \bar{y}_{ab} \right\} = O$$

Equation (3) can be written as follows:

$$\frac{N_A}{n_A} \left\{ p - \frac{(1\text{-}\alpha)\, (\bar{y}_a - p\bar{y}_{ab})}{\sigma^2_{ab}}\, \bar{y}_{ab} \right\}$$

$$+ \frac{N_B}{n_B} \left\{ - q + \frac{(1\text{-}\beta)\, (\bar{y}_b - q\bar{y}_{ab})\, \bar{y}_{ab}}{\sigma^2_{ab}} \right\}$$

$$= \frac{N_A}{n_A} \left\{ p - \frac{(1\text{-}\alpha)\, (\bar{y}_a - p\bar{y}_{ab})\, \bar{y}_{ab}}{\sigma^2_{ab}} \right\}$$

Assuming $\bar{y}_a \doteq \bar{y}_a \doteq \bar{y}_{ab}$, p can be solved from the quadratic formula as follows:

$$p = \frac{-2\sigma^2_{ab} + \bar{y}^2_{ab}\left(3-2-\frac{N_A}{N_B}\right) \pm \bar{y}_{ab}\sqrt{\left(1-\alpha\frac{N_A}{N_B}\right) + \bar{y}^2.\ \left(8-\alpha\left[10-6\frac{N_A}{N_B}\right] + \sigma^2\left[\frac{N^2_A}{N^2_B} + 4\frac{N_A}{N_B}\right]\right\}}}{2\left[y_{ab}(\alpha-1) - \sigma^2_{ab}\right]}$$

If $\frac{N_A}{N_B} = 1$, the quadratic function given above can be expressed as

$$p = \frac{-2\sigma^2_{ab} + 3\bar{y}^2_{ab}(3-3) \pm \bar{y}_{ab}\sqrt{4\sigma^2_{ab}(1-\alpha) + \bar{y}^2_{ab}\ (8-4\alpha\ [10-6] + 5\alpha^2}}{2\left[\bar{y}_{ab}(\alpha-1) - \sigma^2_{ab}\right]}$$

$$= \frac{-2\sigma^2_{ab} + 3\bar{y}^2_{ab}(1-\alpha) \pm \bar{y}_{ab}\sqrt{4\sigma^2_{ab}\ (1-\alpha) + \bar{y}^2_{ab}\ (8-4\alpha) + 5\alpha^2}}{2\left[\bar{y}_{ab}(\alpha-1) - \sigma^2_{ab}\right]}$$

For $\alpha \to 0$

$$p = \frac{-2\sigma^2_{ab} + 3\bar{y}_{ab} \pm \bar{y}_{ab}\sqrt{4\sigma^2_{ab} + 8\bar{y}^2_{ab}}}{-2\bar{y}_{ab} - \sigma^2_{ab}}$$

$$= \frac{-2\sigma^2_{ab} + 3\bar{y}_{ab} \pm 2\bar{y}_{ab}\sqrt{\sigma^2_{ab} + 2\bar{y}^2_{ab}}}{-2\bar{y}_{ab} - \sigma^2_{ab}}$$

For $\alpha \to 1$

$$p = \frac{-2\,\sigma^2_{ab} \pm 2\,\bar{y}_{ab}\sqrt{\sigma^2_{ab} + \bar{y}_{ab}}}{-2\,\sigma^2_{ab}}$$

$$= 1 \pm \frac{2\,\bar{y}_{ab}}{(-2\,\sigma^2_{ab})}\sqrt{\sigma^2_{ab} + \bar{y}_{ab}}$$

$$= 1 - \frac{\bar{y}_{ab}}{\sigma^2_{ab}}\sqrt{\sigma^2_{ab} + \bar{y}_{ab}}$$

For large σ^2_{ab} and small $\bar{y}_{ab}$ and a large number of the units in the overlapped domain in proportion to the number in Frame A this implies to use only Frame A. As the degree of overlap decreases i.e., the proportion $\frac{N_{ab}}{N_A}$, p will decrease in value which means more utilization of Frame B.

Optimum p for a variable cost. A general statement will be made here but the results will be presented in another paper. For the case of two frames, optimization of the function of the variance and cost will yield a cubic function of p. The optimum p is obtained by iteration.

Estimation of Domain Mean (Unconditional or Global Approach)

The estimation of the domain mean will be considered on the basis of the information available on the domain, as follows:

Case 1. No information available on domain

	Number	*Mean*	*Total*
Domain	•	•	•
Overall Domain	N	X	X

Case 2. Partial information available on domain:
Case 2a. Domain mean X_j's are known.
Case 2b. Domain totals X_j's are known.
Case 2c. Domain numbers N_j's are known.

Suppose we define the following variables:

$$w_{ji} = \begin{cases} y_i & \text{if the } i\text{-}^{th} \text{ unit is in the } j\text{-}^{th} \text{ domain} \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{ji} = \begin{cases} x_i & \text{if the } i\text{-}^{th} \text{unit is in the } j\text{-}^{th} \text{ domain} \\ 0 & \text{otherwise.} \end{cases}$$

$$c_{ji} = \begin{cases} 1 & \text{if the } i\text{-}^{th} \text{unit is in the } j\text{-}^{th} \text{ domain} \\ 0 & \text{otherwise.} \end{cases}$$

The first case have been described in detail in an outline paper published in the *Philippine Statistician,* Vol. XVI, Nos. 3-4, 1967, so this paper will put emphasis on the second case of partial information available on the domains.

The estimators used for all subcases in the second situation are the ratio-of-regression-functions type estimators, i.e., $\ddot{y}_j = \frac{u}{v}$, where u is f (x, y) and v is f (x, n_j). So basically our estimators are ratio estimators and possess the properties usually attributable to this family of estimators.

The estimators of domain means under the different subcases are as follows:

$$\bar{y}_{ij.i} = \frac{\bar{w}_j - b_{wz_j} (\bar{z}_j - \bar{x}_j)}{\bar{c}_j - b_{cz_j} (z_{ji} - \bar{x}_j)}$$

$$\text{where } b_{wx_j} = \frac{\sum_{i=1}^{n} (\bar{w}_{ji} - \bar{w}_{j.}) (z_{ji} - \bar{x}_j)}{\sum_{i=1}^{n} (z_{ji} - \bar{x}_j)^2}$$

$$b_{cz_j} = \frac{\sum_{i=1}^{n} (c_{ji} - c_j) (z_{ji} - \bar{x}_j)}{\sum_{i=1}^{n} (z_{ji} - \bar{x}_j)^2}$$

With an approximate variance of

$$V_{\bar{y}_{ij.1}} = \frac{E^2(\bar{w}_{ij.1})}{E^2(\bar{c}_{ij.1})}\left[\frac{N_{\bar{w}_{j.}}}{E^2(\bar{w}_{ij.1})} + \frac{V(\bar{c}_{j.})}{E^2(\bar{c}_{ij.1})} - \frac{2Cov(\bar{w}_{ji}\,\bar{c}_{j.})}{E\bar{w}_{ij.1}\,E\bar{c}_{ij.1}}\right.$$

$$-V_{\bar{z}_{j.}}\left\{\frac{B_{wz_j}}{E^2\bar{w}_{ij.1}} + \frac{B_{cz_j}}{E_{\bar{c}_{ij.1}}} + \frac{2B_{wz_j}B_{cz_j}}{E_{\bar{w}_{ij.1}}\,E_{\bar{c}_{ij.1}}}\right\}$$

$$\left. + \frac{2}{E(\bar{w}_{ij.1})E(\bar{c}_{ij.1})}\left\{B_{wz_j}\,Cov(\bar{c}_{ji}\,\bar{z}_{j.}) + B_{cz_j}\,Cov\,\bar{w}_{j.}\bar{z}_j\right\}\right]$$

Domain totals X_j's are known:

$$\ddot{y}_{ij.z} = \frac{\bar{w}_{j.} - b_{wuj}\,(\bar{u}_j - \bar{U}_j)}{\bar{c}_j - b_{cuj}\,(\bar{u}_j - \bar{U}_j)}$$

where $\bar{U}^j = \frac{J}{N}\,X_j$ and J = the total number of domain $\dot{U}_j = \sum^n x_i / n$,

$$b_{uj} = \frac{\sum_{i=1}^{n}(w_{ji} - \bar{w}_{j.})(z_{ji} - \bar{U}_j)}{\sum_{i=1}^{n}(Z_{ji} - \bar{U}_j)^2}$$

$$b_{cuj} = \frac{\sum_{i=1}^{n}(c_{ji} - \bar{c}_j)(z_{ji} - \bar{U}_j)}{\sum_{i=1}^{n}(z_{ji} - \bar{U}_j)^2}$$

and the approximate variance formula is

$$V(\bar{y}_{ij.2}) \doteq \frac{E^2(\bar{w}_{ij.2})}{E^2(\bar{c}_{ij.2})}\Bigg[\frac{V\bar{w}_{j.}}{E^2(\bar{w}_{ij.2})} + \frac{V\bar{c}_{j.}}{E^2\bar{c}_{ij.2}}$$

$$-\frac{2\mathrm{Cov}\,(\bar{w}_{j.}, \bar{c}_{j.})}{E(\bar{w}_{ij.2})E\bar{c}_{ij.2}} - V(\bar{u}_j)\left\{\frac{B^2_{wuj}}{E^2(\bar{w}_{ij.2})} + \frac{B^2_{cuj}}{E^2\bar{c}_{ij.}}\right.$$

$$\left.-\frac{2B_{wuj}B_{cuj}}{E(\bar{w}_{ij.2})E(\bar{c}_{ij.2})}\right\} + \frac{2}{E(\bar{w}_{ij.2})E(\bar{c}_{ij.2})}$$

$$\left\{B_{wuj.}\,\mathrm{Cov}(\bar{c}_{j.}\bar{u}_{j.}) + B_{cuj}\,\mathrm{Cov}(\bar{w}_{j.}, \bar{u}_j)\right\}\Bigg]$$

Domain numbers N_j's are known:

$$\bar{y}_{ij.3} = \frac{\bar{w}_{j.} - b'_{wuj}(\bar{u}_j - \bar{u}'_j)}{\bar{c}_{j.} - b'_{cuj}(\bar{u}_j - \bar{u}'_j)}$$

where $\bar{u}'_j = \frac{N_j}{N} S\bar{X}$, $\bar{u}_j = \Sigma x_i / n$

$$b'_{wuj} = \sum_{i=1}^{n}(w_{ji} - \bar{w}_{j.})(z_{ji} - \bar{u}'_j) \Big/ \sum_{i=1}^{n}(z_{ji} - \bar{u}_j)^2$$

$$b'_{cuj} = \sum_{i=1}^{n}(c_{ji} - \bar{c}_{j.})(z_{ji} - \bar{u}'_j) \Big/ \sum_{i=1}^{n}(z_{ji} - \bar{u}_j)^2$$

with an approximate variance of

$$V(\bar{y}_{ij.3}) = \frac{E^2(\bar{w}_{ij.3})}{E^2(\bar{c}_{ij.3})}\;\frac{V(\bar{w}_{j.})}{E^2\bar{w}_{ij.3}}\;\frac{V(\bar{c}_{j.})}{E^2\bar{c}_{ij.3}}$$

$$- \frac{2\,\text{Cov}\,(\bar{w}_{j.}, \bar{c}_{j.})}{E(\bar{w}_{ij.3})E(\bar{c}_{ij.c})} - V\,\bar{u}'_{j.}\,\frac{B'^{2}_{wuj.}}{E^{2}(\bar{w}_{ij.3})}$$

$$+ \frac{B'^{2}_{cuj}}{E^{2}(\bar{c}_{ij.3})} + \frac{2\,B'_{wuj}\,B'_{cuj}}{E(\bar{w}_{ij.3})E(\bar{c}_{ij.3})}$$

$$+ \frac{2}{E(\bar{w}_{ij.}3)\;E(c_{ij.}3)}\{B'_{wuj}\,\text{Cov}\,(\bar{c}_{j}, \bar{u}'_{j})$$

$$+ B'_{cuj}\,\text{Cov}\,(\bar{w}_{j.}\bar{u}'_{j.})\}$$

applications of the above estimators are given in the following table:

Case Situation No.	Domain	β_w	β_c	$y_{ij.}$	Var $y_{ij.}$	Cov ($y_{ij.}$, $c_{ij.}$)
1	Tenure of Farm Operator	2.89600	.00013	2,139.19	211,448	–17.76
2a	"	6.10805	.00471	1,948.02	133,181	–10.13
2b	"	12.10259	.01327	1,533.71	92,192	– 0.77
2c	"	2.90616	.00010	2,174.27	288,320	–17.61
1	Livestock Farm	4.08920	–.00044	2,743.42	255,862	– 6.58
2a	"	8.81110	–.00540	4,419.66	–207,093	+18.12
2b	"	1.87569	–.00086	2,547.30	663,120	–89.92
2c	"	0.00201	–.00097	4,849.06	127,946	+33.97
1	Farm Size 100-219	0.690.40	–.00089	2,043.86	116,560	– 6.31
2a	"	0.83040	.00011	1,990.37	117,425	–15.72
2b	"	–3.22550	–.00260	2,037.79	161,521	–15.89
2c	"	4.54470	.00276	1,935.66	17,237	–34.26

Here $\bar{y}_{ij.}$ is the net farm income, which was estimated for different domains of farm operators. The first domain was tenure of farm operator; the second, type of farm; and the third, farm size. The feasibility of using facsimile of the domain mean of the explanatory variables have been demonstrated above. As a matter of fact, it seems that the use of facsimiles will yield more efficient estimators of the domain mean than in the situation when the domain means are known.

Estimation of Domain Means (Conditional or Local Approach)

Another method of estimating the variance of the domain means is to use a conditional approach, i.e.,

$$\text{Var}\ (\bar{y}_{ij.}/n_{j.})$$

The estimator with this approach will be not of the ratio type but of a conditional type estimator, viz., $\bar{y}_{ij.}/n_j$. If $n_{j.}$ and $y_{ij.}$ are independent, the usual method used for a stratified design can be used. Details on this aspect of this study will be discussed in another paper.

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